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Host Organisation	Centrum Wiskunde & Informatica
Scientific coordinator	Prof. Daniel Dadush



I – SCIENTIFIC ACTIVITY DURING YOUR FELLOWSHIP

During my fellowship, I worked at the Networks & Optimization group at CWI, hosted by the group leader Prof. Daniel Dadush.

My research initially focused on finalizing my ongoing project on path decompositions of oriented graphs, together with Viresh Patel from Queen Mary University of London, which is now published in the *European Journal of Combinatorics*. In this project, our focus was on partitioning the edges of directed graphs using as few paths as possible. We worked on a conjecture of Pullman from 1980, which states that every orientation of every regular graph of odd degree can be decomposed into the minimum number of paths dictated by the vertex degree imbalances. This conjecture is a broad generalization of the well-known Kelly's conjecture in the area (settled in 2012 by Daniela Kühn and Deryk Osthus), which asserts that every regular tournament has a Hamilton decomposition. We proved Pullman's conjecture in the random setting; that is, for a uniformly randomly chosen regular graph of odd degree, with high probability, every orientation satisfies the desired property. Although the main work was submitted shortly before starting my fellowship, I worked on the revisions during my time at CWI.

I was fortunate to have the opportunity to participate in the Research Exchange Program hosted by Tomáš Masařík and Michał Włodarczyk at the University of Warsaw. We first discussed several problems online that we could potentially work on together. During my visit, we eventually decided to focus on a problem concerning partitioning the edges of a graph into as few paths as possible, which is one of the classical recurring topics in extremal graph theory. One interesting aspect of this problem is that it is well known that determining the minimum number of paths required to decompose a graph is NP-hard for general graphs, even when we only ask whether two paths suffice. After intensive brainstorming during my one-week visit, we managed to complete the solution after continuing to work on it online afterwards. We first provided a simple and precise formula for the path number of subcubic graphs, which was known to be impossible when the degree is higher. Most importantly, we developed a fixed-parameter tractable algorithm for graphs that are 'close' to being subcubic in terms of edge-deletion distance; that is, the path number of 'almost-subcubic' graphs can be computed efficiently. Our collaboration is now published in a well-established computer science conference, namely *WG 2026*, and we have also submitted it to a high-level graph theory journal.

Through this grant, I also gained experience in developing my own research agenda and working on a problem individually, which is crucial at this stage of my career. During my PhD, together with Allan Lo and Viresh Patel, we had proposed directed and oriented analogues of a problem concerning path partitions of regular graphs. The original problem is closely related to the linear arboricity conjecture, one of the most famous problems in extremal graph theory, and has been proved when the degree is at most six. During my fellowship, I investigated the analogous conjectures in regular directed and oriented graphs. By deploying the discharging method, I managed to prove the directed analogue up to degree four and the oriented analogue up to degree two. I am currently finalizing my paper to submit it.

During my fellowship, I also visited my upcoming academic institution, the School of Mathematics at the Georgia Institute of Technology, to deliver an invited talk at the Combinatorics Seminar. I presented my ongoing work on path partitions in regular digraphs. During the visit, I discussed potential research directions with my prospective supervisor, Tom Kelly. We decided to begin working on a few well-known problems in design theory, including Rodney's conjecture from 1989, which states that every Latin square of order n contains a 2-apex, meaning a set of $2n$ cells that has two cells from each row, each column, and each symbol. Such problems in design theory can be formulated in terms of hypergraphs in a way that the task becomes equivalent to the existence of a perfect matching or a 2-factor in a certain hypergraph. I spent



time reviewing the literature on related problems and learning some of the techniques required for approaching this type of questions, such as absorption and entropy methods.

During my fellowship, I have enjoyed the being part of the combinatorics environment in the Netherlands. I have participated Dutch Days of Combinatorics, both in 2025 and 2026, which is an annual two-days event bringing researchers working in the combinatorics around the Netherlands and even for neighbouring countries. I followed the weekly discrete mathematics seminars in Korteweg de-Vries Institute at University of Amsterdam, which is a neighbour institute of CWI. I enjoyed our biweekly Networks and Optimization group seminars at CWI, and I also gave a talk during my first months here. I also participated several talks in the framework of the workshop Phase Transitions in Combinatorics, Algorithms, and Probability, organized by people from our research group. Through the fellowship, I have attended a few big conferences in my research are across the Europe, including EuroComb'25 which held in Budapest, Combinatorics and Geometry'25 which held in Istanbul, and WG'26 which held in Kortrijk. Apart from these, I continued to be my duties at Turkish Math Olympiad committee and represented my home country in Balkan Mathematical Olympiads in 2025 and 2026 as a deputy team leader. In addition, I contributed to [MATH+ advent calendar 2025](#) with Problem 13 and Problem 17, which consists of 24 mathematical puzzles designed to illustrate that mathematics is fascinating, creative, and useful in a simple yet enjoyable way.

II – PUBLICATION(S) DURING YOUR FELLOWSHIP

Published

- Viresh Patel and **Mehmet Akif Yıldız**, [Path decompositions of oriented graphs](#), European Journal of Combinatorics, 134, 104346 (2026).

(This article was written and submitted prior to the fellowship period, but the revisions and publishing process were completed during the fellowship)

We address the problem of partitioning the edges of a directed graph into as few paths as possible. A directed graph D is said to be consistent if its edges can be decomposed into $\text{ex}(D)$ many paths, where $\text{ex}(D)$ is a natural lower bound for any path decomposition dictated by vertex degree imbalances. As a generalization of Kelly's conjecture on Hamilton decompositions of regular tournaments, Alspach, Mason, and Pullman conjectured in 1976 that every tournament of even order is consistent. This was recently verified for large tournaments by Girão, Granet, Kühn, Lo, and Osthus. A more general conjecture of Pullman states that every orientation of a regular graph of odd degree is consistent. In this paper, we prove that such a statement holds for a random regular graph of odd degree with high probability. Along the way, we also verify Pullman's conjecture for graphs with no short cycles.

- Tomáš Masařík, Michał Włodarczyk, **Mehmet Akif Yıldız**, [Optimal path partitions in subcubic and almost-subcubic graphs](#), In 52nd International Workshop on Graph-Theoretic Concepts in Computer Science (WG 2026), Leibniz International Proceedings in Informatics (LIPIcs), Volume 376, pp. 32:1-32:16 (2026).

We consider the problem of partitioning the edges of a graph into as few paths as possible. This is a subject of the classic conjecture of Gallai and a recurring topic in combinatorics. Regarding the complexity of partitioning a graph optimally, Peroché [Discrete Appl. Math. 1984] proved that it is NP-hard already on graphs of maximum degree four, even when we only ask if two paths suffice. We show that the problem is solvable in polynomial time on subcubic graphs and then we present an efficient algorithm for "almost-subcubic" graphs. Precisely, we prove that the problem is fixed-parameter tractable when parameterized by the edge-deletion distance to a subcubic graph. To this end, we reduce the task to model checking in



first-order logic extended by disjoint-paths predicates (FO+DP) and then we employ the recent tractability result by Schirrmacher, Siebertz, Stamoulis, Thilikos, and Vigny [LICS 2024].

In preparation

- **Mehmet Akif Yıldız**, Path partitions in regular digraphs

The path partition conjecture of Magnant and Martin, which is closely related to the linear arboricity conjecture, asserts that every n -vertex d -regular undirected graph can be partitioned into at most $n/(d+1)$ vertex-disjoint paths. An approximate version of this conjecture was proved by Montgomery, Müyesser, Pokrovskiy, and Sudakov in 2024, and it was verified up to a factor of two by Christoph, Draganić, Girão, Hurley, Michel, and Müyesser in 2025, while its exact form is known to hold when d is at most six (Magnant and Martin 2009; Feige and Fuchs 2022). Motivated by the directed linear arboricity conjecture, it is natural to consider directed and oriented analogues of the path partition conjecture. In this paper, we prove the directed analogue when d is at most four and the oriented analogue when d is two.

III – ATTENDED SEMINARS, WORKHOPS, CONFERENCES

Presentations

- Dutch Days of Combinatorics, *June 16, 2026*
- 52nd Int. Workshop on Graph-Theoretic Concepts in Computer Science (WG), *June 3, 2026*
- Georgia Tech Combinatorics Seminar, *April 24, 2026*
- Combinatorics and Geometry in Istanbul, *August 6, 2025*
- Boğaziçi University Mathematics Colloquium, *July 19, 2025*
- Networks & Optimization Seminar, *April 16, 2025*

Participations

- Phase Transitions in Combinatorics Colloquium, CWI, *April 17, 2026*
- Phase Transitions in Probability Colloquium, CWI, *April 2, 2026*
- European Conf. on Combinatorics, Graph Theory, and App. (EuroComb), *August 25-29, 2025*
- Dutch Days of Combinatorics, *March 25-26, 2025*

IV – RESEARCH EXCHANGE PROGRAMME (REP)

For REP, I visited the Faculty of Mathematics, Informatics and Mechanics at the University of Warsaw between September 21–26, 2025, hosted by Tomáš Masařík and Michał Włodarczyk. During the visit, we first discussed whether Lovász's classical result on path and cycle decompositions of graphs could be strengthened within the framework of 'parameterization above guarantee', an approach in which one starts from a combinatorial theorem guaranteeing a structure of size X and asks whether a structure of size $X+k$ can be found algorithmically. We later decided to focus on a specific fixed-parameter tractability problem concerning path decompositions of sparse graphs. The ideas we developed during the research visit later led to a fruitful collaboration, and the resulting paper has now been published at the WG 2026 conference.